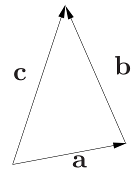
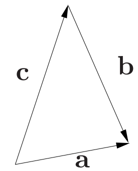
Vektorrechnung

Vektoraddition:



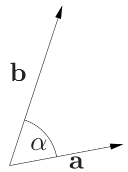
$\mathbf{c} = \mathbf{a} + \mathbf{b}$

$\mathbf{c} = \mathbf{a} - \mathbf{b}$

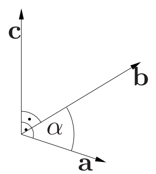


Skalarprodukt: $c = \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| \cdot |\mathbf{b}| \cdot \cos(\alpha) = \mathbf{b} \cdot \mathbf{a}$

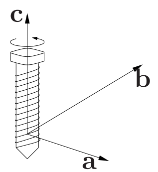
Merke: c ist ein Skalar!



Kreuzprodukt: $\mathbf{c} = \mathbf{a} \times \mathbf{b} = -(\mathbf{a} \times \mathbf{b}) \quad ; \quad |\mathbf{c}| = |\mathbf{a}| \cdot |\mathbf{b}| \cdot \sin(\alpha)$



Richtung von **c** entsprechend der Rechtsschraubenregel.
Merke: **c** ist ein Vektor!



Einheitsvektor: $\mathbf{e}_r = \frac{\mathbf{r}}{|\mathbf{r}|} \quad ; \quad |\mathbf{e}_r| = 1$

Kartesische Vektorkomponenten: $\mathbf{r} = \mathbf{x} + \mathbf{y} + \mathbf{z} = x\mathbf{e}_x + y\mathbf{e}_y + z\mathbf{e}_z$

Vektorbetrag: $r = |\mathbf{r}| = \sqrt{x^2 + y^2 + z^2}$

Rechenregeln:

$\mathbf{a}(\mathbf{b} \times \mathbf{c}) = \mathbf{c}(\mathbf{a} \times \mathbf{b}) = \mathbf{b}(\mathbf{c} \times \mathbf{a})$ $\qquad$ $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b})$

$(\mathbf{a} \times \mathbf{b})(\mathbf{c} \times \mathbf{d}) = (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c})$ $\qquad$ $\mathbf{c} = \mathbf{e}(\mathbf{e} \cdot \mathbf{c}) - \mathbf{e} \times (\mathbf{e} \times \mathbf{c})$

Koordinatensysteme

Kartesische Koordinaten (x, y, z): $d\mathbf{s} = \mathbf{e}_x dx + \mathbf{e}_y dy + \mathbf{e}_z dz \quad ; \quad dV = dxdydz$

Zylinderkoordinaten (r, ϕ, z):

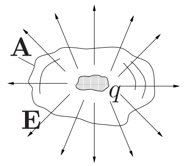
$x = r \cdot \cos(\phi)$	$0 \leq r < +\infty$	$d\mathbf{s} = \mathbf{e}_r dr + r \cdot \mathbf{e}_\phi d\phi + \mathbf{e}_z dz$
$y = r \cdot \sin(\phi)$	$0 \leq \phi \leq 2\pi$	$dV = rdrd\phi dz$
$z = z$	$-\infty < z < +\infty$	

Kugelkoordinaten (r, θ, ϕ):

$x = r \cdot \sin(\theta) \cdot \cos(\phi)$	$0 \leq r < +\infty$	
$y = r \cdot \sin(\theta) \cdot \sin(\phi)$	$0 \leq \phi \leq 2\pi$	$d\mathbf{s} = \mathbf{e}_r dr + r \cdot \mathbf{e}_\theta d\theta + r \cdot \sin(\theta)\, \mathbf{e}_\phi d\phi$
$z = r \cdot \cos(\theta)$	$0 \leq \theta \leq \pi$	$dV = r^2 \sin(\theta)\, drd\theta d\phi$

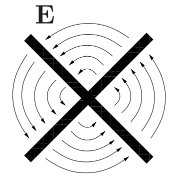
Elektrostatik

Elektrostatische Gleichungen: mit $\mathbf{B} = 0$ (Elektro-) und $\frac{d}{dt} = 0$ (-statik)



$$\oint_A \mathbf{E} d\mathbf{A} = \frac{q}{\epsilon}$$

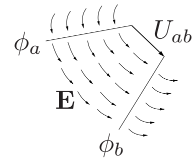
$$\oint_\Gamma \mathbf{E} d\mathbf{s} = 0$$



Elektrisches Feld E, Potential ϕ und elektrische Spannung U:

$$\phi(x) = -\int_0^x \mathbf{E} d\mathbf{s}$$

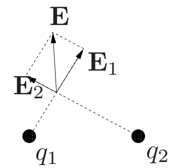
$$U_{ab} = \phi_a - \phi_b = -\int_b^a \mathbf{E} d\mathbf{s} = \int_a^b \mathbf{E} d\mathbf{s}$$



Überlagerung (Superposition) von Feldern:

$$\mathbf{E}(x,y,z) = \mathbf{E}_1(x,y,z) + \mathbf{E}_2(x,y,z)$$

$$\phi(x,y,z) = \phi_1(x,y,z) + \phi_2(x,y,z)$$



Potentielle Energie einer Ladung: $W_{pot} = q \cdot \phi$

Elektrostatik bei vorhandener Materie:

Elektrisches Feld **E** :

$$\int_A \mathbf{E} d\mathbf{A} = \frac{q_{frei} + q_{pol}}{\epsilon_0}$$

Verschiebungsdichte **D** :

$$\int_A \mathbf{D} d\mathbf{A} = q_{frei}$$

Polarisation **P**:

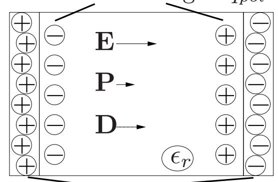
$$|P| = \sigma_{pol} = \frac{q_{pol}}{A}$$

P von $-\sigma_{pol}$ nach $+\sigma_{pol}$

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon_r \epsilon_0 \mathbf{E} = \epsilon \mathbf{E}$$

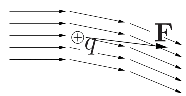
$$\epsilon = \epsilon_r \epsilon_0 = (1 + \chi)\, \epsilon_0$$

Polarisationsladungen q_{pol}



freie Ladungen q_{frei}

Achtung!
Der Ansatz, die Materialeigenschaften mit den Parametern ϵ_r und χ zu beschreiben, ist eine Näherung, die nur in linearen, isotropen Materialien gilt!

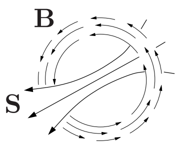


Kraft auf Ladung (Coulombkraft):

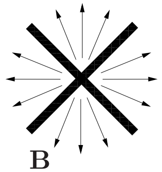
$$\mathbf{F} = q \cdot \mathbf{E}$$

Magnetostatik

Magnetostatische Gleichungen: mit $\mathbf{E} = 0$ (Elektro-) und $\frac{d}{dt} = 0$ (-statik)



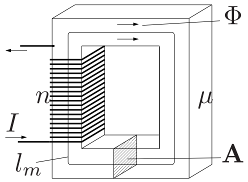
$$\frac{1}{\mu_0}\oint_\Gamma \mathbf{B}d\mathbf{s} = \int_A \mathbf{S}d\mathbf{A} \qquad \oint_A \mathbf{B}d\mathbf{A} = 0$$

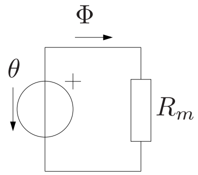


Magnetischer Kreis:

$$\Phi = \frac{\theta}{R_m}$$

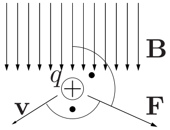
mit $R_m = \frac{l_m}{\mu \cdot A}$ und $\theta = n \cdot I$





Kraft auf Ladung (Lorentzkraft):

$$\mathbf{F} = q \cdot (\mathbf{v} \times \mathbf{B}) = I \cdot (\mathbf{s} \times \mathbf{B})$$



Magnetostatik bei vorhandener Materie:

Magnetisches Feld $\mathbf{H}$:

$$\oint_\Gamma \mathbf{H}d\mathbf{s} = \int_A \mathbf{S}_S d\mathbf{A}$$

Flussdichte $\mathbf{B}$:

$$\frac{1}{\mu_0}\oint_\Gamma \mathbf{B}d\mathbf{s} = \int_A (\mathbf{S}_S + \mathbf{S}_m)\, d\mathbf{A}$$

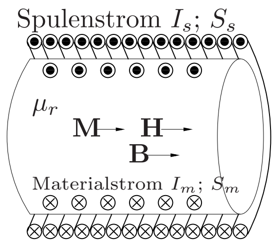
$$\int_A \mathbf{D}d\mathbf{A} = q_{frei}$$

Magnetisierung $\mathbf{M}$:

$$\oint_\Gamma \mathbf{M}d\mathbf{s} = \int_A \mathbf{S}_m d\mathbf{A}$$

$$\mathbf{B} = \mu_0\,(\mathbf{H} + \mathbf{M}) = \mu_r\mu_0\mathbf{H} = \mu\mathbf{H}$$

$$\mu = \mu_r\mu_0 = (1+\chi)\,\mu_0$$



Achtung!
Der Ansatz, die Materialeigenschaften mit den Parametern μ_r und χ zu beschreiben, ist eine Näherung, die nur in linearen, isotropen Materialien gilt!

Energie im Magnetischen Feld:

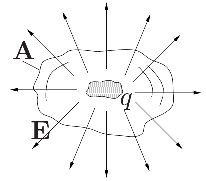
Ohne Materie: $W_m = \frac{1}{\mu_0}\int_V B^2 dV$

allgemein: $E_m = \int_V \int_0^{\mathbf{B}} \mathbf{H}d\mathbf{B}dV$

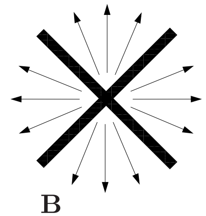
Quasistationärer Zustand

Maxwell'sche Feldgleichungen:

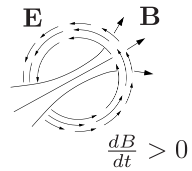
$$\oint_A \mathbf{E}d\mathbf{A} = \frac{q}{\epsilon_0}$$



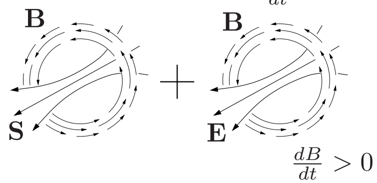
$$\oint_A \mathbf{B}d\mathbf{A} = 0$$



$$\oint_\Gamma \mathbf{E}d\mathbf{s} = -\frac{\partial}{\partial t}\int_A \mathbf{B}d\mathbf{A}$$

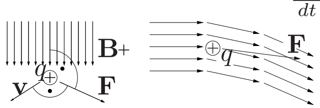


$$\frac{1}{\mu_0}\oint_\Gamma \mathbf{B}d\mathbf{s} = \int_A \left(\mathbf{S} + \epsilon_0 \frac{d}{dt}\mathbf{E}\right) d\mathbf{A}$$



Lorentz-Coulomb-Beziehung:

$$\mathbf{F} = q \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

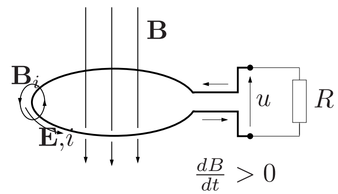


Lenz'sche Regel:

$$\frac{d}{dt}\mathbf{B} \Rightarrow \mathbf{E} \Rightarrow i \Rightarrow \mathbf{B}_i$$

$$\text{mit: } \mathbf{B}_i \uparrow\downarrow \frac{d}{dt}\mathbf{B}$$

Richtung von u entsprechend i (Erzeuger-Zählpfeilsystem)



Biot-Savart'sches Gesetz:

$$\mathbf{B}\,(I) = \frac{\mu_0 I}{4\pi}\int_l \frac{d\mathbf{l} \times \mathbf{r}_{12}}{r_{12}^3}$$